Distributed Spectrum Management and Relay Selection in Interference-limited Cooperative Wireless Networks

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Outline

1. Introduction
2. Related Work
3. Problem Formulation
4. Proposed Solution Algorithm
5. Performance Analysis
6. Conclusions
Emerging multimedia services require high data rate
Need to maximize transport capacity of wireless networks
Introduction

Increase transport capacity by leveraging **frequency and spatial diversity**

- **Dynamic spectrum access**: improve spectral efficiency (frequency diversity)
- **Cooperative communications**: enhance link connectivity (spatial diversity)
Introduction

Increase transport capacity by leveraging **frequency and spatial diversity**

- **Dynamic spectrum access**: improve spectral efficiency (frequency diversity)
- **Cooperative communications**: enhance link connectivity (spatial diversity)

**Open challenge**: Distributed control strategies

- to dynamically jointly assign portions of spectrum and cooperative relays
- to maximize network-wide data rate
- in interference-limited infrastructure-less networks
Centralized control in interference-free networks

Related Work – Leveraging Spectral And Spatial Diversity

- **Centralized control in interference-free networks**

- **Distributed control in interference-free networks**
Centralized control in interference-free networks

Distributed control in interference-free networks

Centralized control in interference-limited networks
Centralized control in interference-free networks

Distributed control in interference-free networks

Centralized control in interference-limited networks

We focus on distributed control in interference-limited infrastructure-less networks
System Model

- **Interference-limited infrastructure-less cooperative network**
  - Uncoordinated source-destination pairs
  - Each source transmits using direct link or through cooperative relaying
  - Dynamically access a portion of spectrum to avoid interference

- **Assumptions**
  - Single hop (no layer-3 routing)
  - Each source uses at most one relay
  - Each relay can be used by at most one source
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Problem Formulation – Overall Model

Objective

Maximize sum utility (capacity, log-capacity) of multiple concurrent traffic sessions
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Maximize sum utility (capacity, log-capacity) of multiple concurrent traffic sessions

By Jointly Optimizing
- Relay selection (whether to cooperate or not, and through which relay)
- Dynamic spectrum access (which channel(s) to transmit on, and at what power)
Problem Formulation – Overall Model

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| **By Jointly Optimizing** | Relay selection (whether to cooperate or not, and through which relay)  
Dynamic spectrum access (which channel(s) to transmit on, and at what power) |
| **Subject to** | Total power constraint  
Relay selection constraint |
Problem Formulation – Link Capacity Model

Cooperative Transmission (Decode-and-Forward) [1]

\[ C_{\text{cop}}^{s, r, f} = \frac{B_f}{2} \min(\log_2(1 + \text{SINR}_{s2r}^s,r,f), \log_2(1 + \text{SINR}_{s2d}^s,s,f + \text{SINR}_{r2d}^r,s,f)) \]

- Choices of relay node and transmit power are important!

### Problem Formulation – Link Capacity Model

#### Cooperative Transmission (Decode-and-Forward) [1]

\[
C^{s,r,f}_{cop} = \frac{B_f}{2} \min \left( \log_2 \left(1 + \text{SINR}^{s,r,f}_{s2r}\right), \log_2 \left(1 + \text{SINR}^{s,s,f}_{s2d} + \text{SINR}^{r,s,f}_{r2d}\right) \right)
\]

– Choices of relay node and transmit power are important!

#### Direct Transmission

\[
C^{s,f}_{dir} = B \log_2 \left(1 + \text{SINR}^{s,s,f}_{s2d}\right)
\]

– Capacity of cooperative transmission may be higher or lower than that of direct transmission. **Cooperate or not?**

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Problem Formulation – Mixed Integer Non-Convex Problem

Maximize

\[ U = \sum_{s \in S} U_s(P, Q, \alpha) \rightarrow \text{Utility function: } \log(C_s) \]

Subject to

\[ \alpha_r^s \in \{0, 1\}, \forall s \in S, \forall r \in R \rightarrow \text{Integer, } 1: \text{selected, } 0: \text{not} \]

\[ \sum_{r \in R} \alpha_r^s \leq 1, \forall s \in S \rightarrow \text{Each session uses at most one relay} \]

\[ \sum_{s \in S} \alpha_r^s \leq 1, \forall r \in R \rightarrow \text{Each relay selected by at most one session} \]

\[ P_s^f \geq 0, \forall s \in S, \forall f \in F \rightarrow \text{Power allocation for source, real, nonnegative} \]

\[ Q_r^f \geq 0, \forall r \in R, \forall f \in F \rightarrow \text{Power allocation for relay, real, nonnegative} \]

\[ \sum_{f \in F} P_s^f \leq P_{s_{\text{max}}}, \forall s \in S \rightarrow \text{Power budget of source} \]

\[ \sum_{f \in F} Q_r^f \leq Q_{r_{\text{max}}}, \forall r \in R \rightarrow \text{Power budget for relay} \]

- Link capacity \( C_s \) is function of SINR
- SINR is nonlinear and non-convex with respect to \( P, Q \) and \( \alpha \)
Proposed Solution Algorithm

MINCoP

- **NP-HARD** in general
Proposed Solution Algorithm

MINCoP

NP-HARD in general

Contributions
Proposed Solution Algorithm

MINCoP

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Contributions

- Branch & Bound / RLT
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Contributions

Branch & Bound / RLT \rightarrow Globally Optimal Solution
Proposed Solution Algorithm

MINCoP

- **NP-HARD** in general

**Contributions**

- Branch & Bound / RLT
- Decompose MINCoP into DSM / DRS
- Globally Optimal Solution
MINCoP

- NP-HARD in general

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MINCoP

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Contributions

- Branch & Bound / RLT
- Decompose MINCoP into DSM / DRS
- Analyze Convergence to NE Through Variational Inequality (VI)
- Globally Optimal Solution
- Propose Iterative Distributed Algorithms
MINCoP

- **NP-HARD** in general

**Contributions**

- Branch & Bound / RLT
- Globally Optimal Solution
- Decompose MINCoP into DSM / DRS
- Propose Iterative Distributed Algorithms
- Analyze Convergence to NE Through Variational Inequality (VI)
- Analyze Price of Anarchy by Comparing to Global Optimum
Globally Optimal Algorithm

Central Idea

- Based on a combination of branch-and-bound (B&B) and convex relaxation.
  - B&B: Iteratively partition the original MINCoP problem into a series of subproblems
  - Convex Relaxation: Relax each subproblem to be convex
Globally Optimal Algorithm – Basic Steps
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![Diagram showing Original MINCoP and Global Optimal Solution]
Globally Optimal Algorithm – Basic Steps

Original MINCoP → Convex Relaxation → Convex Optimization Problem (COP) → Global Optimal Solution
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Original MINCoP → Convex Relaxation → Convex Optimization Problem (COP) → Solve COP → Global Upper Bound → Global Optimal Solution
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Original MINCoP \( \xrightarrow{\text{Convex Relaxation}} \) Convex Optimization Problem (COP) \( \xrightarrow{\text{Solve COP}} \) Global Upper Bound

- Global Optimal Solution
- Search for a Feasible Solution (SFS)

Global Lower Bound
Globally Optimal Algorithm – Basic Steps

Original MINCoP → Convex Relaxation → Convex Optimization Problem (COP) → Solve COP → Global Upper Bound
Global Optimal Solution → Search for a Feasible Solution (SFS) → Global Lower Bound

Sub-MINCoP 1 → Convex Relaxation → COP 1

Sub-MINCoP 2 → Convex Relaxation → COP 2
Globally Optimal Algorithm – Basic Steps

1. **Original MINCoP**
   - Convex Relaxation
   - Convex Optimization Problem (COP)
   - Solve COP

2. **Sub-MINCoP 1**
   - Convex Relaxation
   - COP 1
   - Solve COP

3. **Sub-MINCoP 2**
   - Convex Relaxation
   - COP 2

4. **Global Upper Bound**
5. **Global Lower Bound**
6. **Global Optimal Solution**
7. **Search for a Feasible Solution (SFS)**
Globally Optimal Algorithm – Basic Steps

Original MINCoP

Convex Relaxation

Convex Optimization Problem (COP)

Solve COP

Global Upper Bound

Global Optimal Solution

Search for a Feasible Solution (SFS)

Global Lower Bound

Sub-MINCoP 1

Convex Relaxation

COP 1

Solve COP

Sub-MINCoP 2

Convex Relaxation

COP 2

SFS
Globally Optimal Algorithm – Basic Steps

- Original MINCoP → Convex Relaxation → Convex Optimization Problem (COP) → Solve COP → Global Upper Bound
  → Global Lower Bound
  → Search for a Feasible Solution (SFS)

- Sub-MINCoP 1 → Convex Relaxation → COP 1 → Solve COP → New Global Upper Bound
  → New Global Lower Bound

- Sub-MINCoP 2 → Convex Relaxation → COP 2
Example: Reduction of Feasible Set
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![Diagram showing the reduction of feasible set through Convex Relaxation](image-url)
Example: Reduction of Feasible Set

Upper bound of Sub-MINC\textup{co}P 2 is smaller than lower bound of Sub-MINC\textup{co}P 1 - Sub-MINC\textup{co}P 2 pruned
Example: Reduction of Feasible Set

Upper bound of Sub-MINCoP 2 is smaller than lower bound of Sub-MINCoP 1  
Sub-MINCoP 2 pruned
Example: Reduction of Feasible Set
VI to facilitate theoretical analysis

- Hard to obtain global optimum in distributed way
- Design algorithms to achieve Nash Equilibrium
- Nash Equilibrium analysis is challenging due to complicated expression of utility functions
- Variational Inequality Theory [2]
  - Broader applicability than classical game theory results
  - Well developed tools for existence and convergence analysis
  - Applies to our problem under certain conditions

Centralized Optimal Solution

Distributed Algorithms
Nash Equilibrium

- Concept from noncooperative game theory
- At Nash Equilibrium no user has incentive to deviate from current transmission strategy
- $x_i$: Transmission strategy of player $i$
- $x_{-i}$: Transmission strategy of all other players except $i$
- Nash Equilibrium problem is defined to find $x^*$ such that

$$x_i^* = \arg \max_{x \in \mathcal{Q}_i} f_i(x_i, x_{-i}^*), \ \forall i$$
Proposed Solution Algorithm – Nash Equilibrium & VI

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Variational Inequality (VI)

- Generalization of optimization and game theory
  $$\langle x - x^* \rangle^T F(x^*) \geq 0, \forall x \in X$$
  
  $F$: Vector of gradient functions of utility function
Proposed Solution Algorithm – Nash Equilibrium & VI

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Variational Inequality (VI)

- Generalization of optimization and game theory
- $(x - x^*)^T F(x^*) \geq 0, \forall x \in X$
- $F$: Vector of gradient functions of utility function
- Each solution of VI is a Nash Equilibrium
Proposed Solution Algorithm – Challenges With VI

**Monotonicity**

- VI theory requires mapping function $F$ to be at least component-wise monotonic

\[ F = (\nabla_{x_s} U_s)_{s \in S} \rightarrow \text{Vector of gradient of utility function} \]

- Hard for simultaneous optimization of spectrum allocation and relay selection
Proposed Solution Algorithm – Challenges with VI

Monotonicity
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Differentiability
- Utility function is not smooth due to $\min(\cdot)$ operation when DF is used
**Proposed Solution Algorithm – Challenges With VI**

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| **Differentiability** | Utility function is not smooth due to min($\cdot$) operation when DF is used |

| **Decomposability** | Relay selection variables are coupled with each other |
|                    | Domain set of a player is function of transmission strategy of other players, hence not fixed |
|                    | Resulting Nash Equilibrium problem or VI problem is more complicated |
Proposed Solution Algorithm – Monotonicity

- Decompose original problem into two subproblems
- Design distributed algorithm for each subproblem
- Perform two algorithms iteratively
- Monotonicity condition is easily satisfied by each subproblem

![Proposed Solution Algorithm Diagram]
### Proposed Solution Algorithm – Differentiability

#### Non-smooth Function

\[ C_{cop}^{s,r,f} = \min(C_{s2r}^{s,r,f}, C_{sr2d}^{s,r,f}) \]

#### Approximation Function

- Approximate \( \min(\cdot, \cdot) \) based on \( \ell_p \)-norm function

\[
\hat{C}_{cop}^{s,r,f} = \ell_p^{-1} ((C_{s2r}^{s,r,f})^{-1}, (C_{sr2d}^{s,r,f})^{-1})
\]

\[
= \left\{ \left[ \left( \frac{1}{C_{s2r}^{s,r,f}} \right)^P + \left( \frac{1}{C_{sr2d}^{s,r,f}} \right)^P \right]^{\frac{1}{P}} \right\}^{-1}
\]

#### Lemmas

- Approximation function is continuously differentiable
- Approximation function is concave when SINR is not too low
Approximation function (smooth) can approximate the original min (non-smooth) with arbitrary precision
**Proposed Algorithm – Convergence of DSM Algorithm**

**Lemma**

- Game of DSM can be reformulated as a VI problem $\text{VI}(\mathcal{X}, F)$

\[
U_s(x_s, x_{-s}) = \log(C_s(x_s, x_{-s})) \rightarrow \text{Utility function}
\]

\[
F = (\nabla_{x_s} U_s)_{s \in S} \rightarrow \text{Vector of gradient of utility function}
\]

\[
\mathcal{X} = \prod_{s \in S} \mathcal{X}_s \rightarrow \text{Cartesian product of domain sets}
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- There exists at least one solution for $\text{VI}(\mathcal{X}, F)$ (also a Nash Equilibrium)
Proposed Algorithm – Convergence of DSM Algorithm

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Theorem

If any two sessions are located sufficiently far away from each other, then a Gauss-Seidel scheme based on the local best response converges to a VI solution, therefore to a Nash Equilibrium.
Proposed Algorithm – Convergence of DSM Algorithm

Do not converge - Strong Interference (much coupling)

Converge - Weak Interference (less coupling)
Proposed Algorithm – Decomposability

- Dynamic relay selection (DRS) as a game
  - Each session is a player → Maximize $U_s(\alpha_s, \alpha_{-s})$
  - Relay selection variables are coupled with each other

\[ \sum_{s \in S} \alpha_r^s \leq 1, \forall r \in R \]

- Resulting joint domain set cannot be decomposed as Cartesian product of multiple sub-domains

- Nash Equilibrium problem with coupled domain sets is called Generalized Nash Equilibrium problem (GNE)

**Lemma**

Resulting GNE can be reformulated as a VI, called QVI, and there exists at least one VI solution which is also a Nash Equilibrium solution.
Proposed Solution Algorithm – DRS

Theorem

The following penalized iterative algorithm converges to a VI solution, which is also a Nash Equilibrium solution [3].

\[ \hat{U}_s(\alpha_s, \alpha_{-s}) = U_s(\alpha_s, \alpha_{-s}) \]

\[ -\frac{1}{2\rho_k} \sum_{r \in \mathcal{R}} \left( \max \left( 0, u_r^k + \rho_k \left( \sum_{s \in \mathcal{S}} \alpha_s r - 1 \right) \right) \right)^2 \]

Penalization

\[ \rho_{k+1} = \rho_k + \Delta \rho, \]

\[ u_{k+1} = \max \left( 0, u_r^k + \rho_k \left( \sum_{s \in \mathcal{S}} \alpha_{s,r}^k - 1 \right) \right) \]

Performance Analysis – System Setup

**System Parameters**
- A terrain of $1500 \text{ m} \times 1500 \text{ m}$
- Session number: 2, 3, 5, 10, 10
- Relay number: 10, 5, 5, 5, 5
- Channel number: 4, 5, 5, 5, 2
- Channel gain: $G_{mn} = d^{-\gamma}(m, n)$
- Path loss factor: $\gamma = 4$
- Average AWGN noise power: $10^{-7} \text{ mW}$
Both DRS and DSM converge fast
Iteration of DRS and DSM converges fast in practice
### Performance Analysis – Price of Anarchy

- $\varepsilon = 95\%$: Centralized algorithm achieves at least 95% of the global optimum
- Distributed algorithm can achieve a performance close to the optimum within several percentages
Conclusions

- Formulation of joint dynamic spectrum allocation and relay selection in interference-limited infrastructure-less networks
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- Developed centralized algorithm to obtain globally optimal solution of MINCoP - NP-HARD
- Designed distributed algorithms by decomposing MINCoP in two subproblems
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Future Work: Implement distributed algorithm in USRP2/GNU Radio testbed
Thanks for your attention
Problem Formulation – Interference Model

- Interference depends on power allocation, relay selection, network scheduling
- Average-based model is used for tractability

\[ I = \frac{1}{2} (I_{\text{time}_{-}\text{slot}_{1}} + I_{\text{time}_{-}\text{slot}_{2}}) \]

- Experiment verified its negligible impact on the overall network performance
Comparison between the average-based interference model and exact interference in synchronization-based cooperative network

- Capacity ratio: $\frac{C_{avg}}{C_{rea}}$
- $C_{avg}$: Capacity calculated using the average-based interference model
- $C_{rea}$: Capacity in reality
- The average-based approximation approximates reality very well
Proof of Convergence of DSM

- Domain set $\mathcal{X}$ is closed and convex
- Mapping function $F_s$ is strongly monotonic
- "Sufficiently far away" is a sufficient condition
- Every session uses a relay, session $s$ uses relay node $r$
- Gradient vector of session $s$ with respect to $x_s$

$$J_{x_s}(U_s) = \left( \left( \frac{\partial U_s}{\partial P^f_{s}} \right)_{f=1}^F, \left( \frac{\partial U_s}{\partial Q^f_{r}} \right)_{f=1}^F \right)$$

- Define a matrix $[\gamma]_{ij}$ as

$$[\gamma]_{sg} \triangleq \begin{cases} \alpha_{s}^{\min}, & \text{if } s = g, \\ -\beta_{sg}^{\max}, & \text{otherwise,} \end{cases}$$

- $\alpha_{s}^{\min} \triangleq \inf_{x \in \mathcal{X}} \lambda_{\text{least}}(J_{x_s}x_s(U_s))$ and $\beta_{sg}^{\max} \triangleq \sup_{x \in \mathcal{X}} \| J_{x_g}x_s(U_g) \|$ 

- $\lambda_{\text{least}}(A)$ is the eigenvalue of $A$ with the smallest absolute value
- Sufficient far away implies that $[\gamma]_{sg}$ is a P-matrix
Proof of Convergence of DRS

- Sufficient to show
  - DRS converges
  - Every accumulation point corresponds to a VI solution
- According to Theorem 3 in [3], max \((0, u_k^r + \rho_k (\sum_{s \in S} \alpha_r^s - 1))\) is bounded
- Penalization item tends to zero as \(\rho_k\) tends to infinity
- According to Theorem 3 in [3], every accumulation point corresponds to a VI solution

Proof of Concavity

- Approximation function $\hat{C}_{cop}^{s,r,f}$ is monotonically increasing
- Domain set $\mathcal{X}$ in the VI problem $\text{VI}(\mathcal{X}, F)$ is bounded
- Only need to show $\hat{C}_{cop}^{s,r,f}$ is a concave function
- A function is concave if it is concave when restricted to any line in the domain
Proposed Solution Algorithm – Practical Issues

- Application Scenarios
  - Multiple co-existing pre-established source-destinations
  - Independent set of transmissions with primary interference constraints
- Dynamic spectrum access
  - SINR measurement is needed at destination and relay nodes, or at source node via control information
  - Cooperative MAC protocol is desired, e.g., CoCogMAC [4]
- Dynamic relay selection
  - Relay periodically broadcasts a “price” frame to claim its price